



# QCD sum rules for the production of the $X(3872)$ as a mixed molecule-charmonium state in $B$ meson decay

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## ABSTRACT

We use QCD sum rules to calculate the branching ratio for the production of the meson  $X(3872)$  in the decay  $B \rightarrow X(3872)K$ , assumed to be a mixture between charmonium and exotic molecular  $[c\bar{q}][q\bar{c}]$  states with  $J^{PC} = 1^{++}$ . We find that in a small range for the values of the mixing angle,  $5^\circ \leq \theta \leq 13^\circ$ , we get the branching ratio  $\mathcal{B}(B \rightarrow XK) = (1.00 \pm 0.68) \times 10^{-5}$ , which is in agreement with the experimental upper limit. This result is compatible with the analysis of the mass and decay width of the mode  $J/\psi(n\pi)$  and the radiative decay mode  $J/\psi\gamma$  performed in the same approach.

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## 1. Introduction

The  $X(3872)$  state has been first observed by the Belle Collaboration in the decay  $B^+ \rightarrow X(3872)K^+ \rightarrow J/\psi\pi^+\pi^-K^+$  [1], and was later confirmed by CDF, D0 and BaBar [2]. This was the first state of an increasing number of candidates for exotic hadrons discovered recently. The current world average mass is  $m_X = (3871.4 \pm 0.6)$  MeV, and the width is  $\Gamma < 2.3$  MeV at 90% confidence level. BaBar Collaboration reported the radiative decay mode  $X(3872) \rightarrow \gamma J/\psi$  [3,4], which determines  $C = +$ . Further studies from Belle and CDF that combine angular information and kinematic properties of the  $\pi^+\pi^-$  pair, strongly favors the quantum numbers  $J^{PC} = 1^{++}$  or  $2^{-+}$  [3,5,6]. Although the new BaBar result favors the  $J^{PC} = 2^{-+}$  assignment [7], established properties of the  $X(3872)$  are in conflict with this assignment [8,9]. Therefore, in this work we will consider the  $X(3872)$  as being a  $J^{PC} = 1^{++}$  state. BaBar Collaboration reported the upper limit of the branching ratio for the production in  $B$  meson decay [10]:

$$\mathcal{B}(B^\pm \rightarrow K^\pm X(3872)) < 3.2 \times 10^{-4}. \quad (1)$$

Recently, Belle Collaboration presented the most precise measurement of the branching fraction  $\mathcal{B}(B^\pm \rightarrow X(3872)K^\pm)\mathcal{B}(X(3872) \rightarrow \gamma J/\psi) = (1.78_{-0.44}^{+0.48} \pm 0.12) \times 10^{-6}$  [11].

The decay modes of the  $X(3872)$  into  $J/\psi$  and other charmonium states indicate the existence of a  $c\bar{c}$  in its content. However the attempts to classify the state in the charmonium spectrum have to deal with the fact that the mass of the  $X(3872)$  is

not compatible with any of the possible candidates in the quark model [12]. Another problem comes from the measurement of the decay rates of the processes  $X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0$  and  $X(3872) \rightarrow J/\psi\pi^+\pi^-$ , which are comparable [3],

$$\frac{X \rightarrow J/\psi\pi^+\pi^-\pi^0}{X \rightarrow J/\psi\pi^+\pi^-} = 1.0 \pm 0.4 \pm 0.3, \quad (2)$$

that could indicate a strong isospin and G parity violation, which is incompatible with a  $c\bar{c}$  structure.

The coincidence between the  $X(3872)$  mass and the  $D^{*0}D^0$  threshold:  $M(D^{*0}D^0) = (3871.81 \pm 0.36)$  MeV [13], inspired the proposal that the  $X(3872)$  could be a molecular ( $D^{*0}D^0 - \bar{D}^{*0}D^0$ ) bound state with small binding energy [14,15]. In particular, considering the  $X(3872)$  as an admixture of neutral and charged components of molecules, i.e.  $D^0\bar{D}^{*0}$  and  $D^+D^{*-}$ , the strong isospin violation observed in Eq. (2) could be explained in a very natural way [16,17]. There is also a possibility that the observed ratio of the  $X$  decaying into  $J/\psi + 2\pi$  or  $3\pi$  may not come from a large isospin breaking. In Ref. [18] the isospin breaking is investigated in the dynamical generation of the  $X$  as a molecular state, and it is found to be small. But, considering that the two pion and three pion states comes from the decays of  $\rho$  and  $\omega$  mesons, the small isospin breaking is compensated by the larger phase space of the  $\rho$  meson, thus explaining the experimental data.

Another interesting interpretation for the  $X(3872)$  is that it could be a compact tetraquark state [19–21].

The co-existence of both  $c\bar{c}$  and multi-quark components is subject of debate in many works, and it is supported by some experimental data. In Ref. [22], a simulation for the production of a bound  $D^0\bar{D}^{*0}$  state with binding energy as small as 0.25 MeV, reported a production cross section that is an order of magnitude

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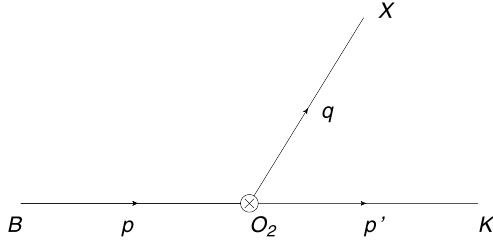


Fig. 1.

smaller than the cross section obtained from the CDF data. A similar result was obtained in Ref. [23] in a more phenomenological analysis. However, as pointed out in Ref. [24], a consistent analysis of the  $D^0 \bar{D}^{*0}$  molecule production requires taking into account the effect of final state interactions of the  $D$  and  $D^*$  mesons.

Besides, the recent observation, reported by BaBar [25], of the decay  $X(3872) \rightarrow \psi(2S)\gamma$  at a rate:  $\frac{\mathcal{B}(X \rightarrow \psi(2S)\gamma)}{\mathcal{B}(X \rightarrow \psi\gamma)} = 3.4 \pm 1.4$ , it is much bigger than the molecular prediction [26]:  $\frac{\Gamma(X \rightarrow \psi(2S)\gamma)}{\Gamma(X \rightarrow \psi\gamma)} \sim 4 \times 10^{-3}$ .

In the framework of the QCD sum rules (QCDSR) the mass of the  $X$  were computed with good agreement with data, considering tetraquark [27] and molecular structures [17]. The same success is not found in decay widths calculations. In Ref. [28] the decay width of the modes  $J/\psi(n\pi)$  are calculated for the tetraquark structure, and the result is one order larger than the total width. In Refs. [29,30] the QCDSR approach was used to study the  $X(3872)$  structure including the possibility of the mixing between two and four-quark states, where it was successfully applied to obtain the mass of the state and the decays widths for the modes  $J/\psi(n\pi)$  and the radiative decay mode  $J/\psi\gamma$ . This was implemented following the prescription suggested in [31] for the light sector. The mixing is done at the level of the currents and is extended to the charm sector. In a different context (not in QCDSR), a similar mixing was suggested already some time ago by Suzuki [23]. Physically, this corresponds to a fluctuation of the  $c\bar{c}$  state where a gluon is emitted and subsequently splits into a light quark-antiquark pair, which lives for some time and behaves like a molecule-like state

The models for the quark structure can also be applied to study the production of the state in  $B$  decays. This subject is studied in different approaches in Refs. [32–34]. In this work we will focus on production of the  $X(3872)$ , using the mixed two-quark and four-quark prescription of Refs. [29,30] to perform a QCDSR analysis of the process  $B^\pm \rightarrow X(3872)K^\pm$ .

## 2. The decay $B \rightarrow X(3872)K$

The process  $B \rightarrow X(3872)K$  occurs via weak decay of the  $b$  quark, while the  $u$  quark is a spectator. The  $X$  meson as a mixed state of molecule and charmonium interacts via  $c\bar{c}$  component of the weak current. In effective theory, at the scale  $\mu \sim m_b \ll m_W$ , the weak decay is treated as a four-quark local interaction described by the effective Hamiltonian (see Fig. 1):

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left[ \left( C_2(\mu) + \frac{C_1(\mu)}{3} \right) \mathcal{O}_2 + \dots \right], \quad (3)$$

where  $V_{ik}$  are CKM matrix elements,  $C_1(\mu)$  and  $C_2(\mu)$  are short distance Wilson coefficients computed at the renormalization scale  $\mu \sim \mathcal{O}(m_b)$ . The four-quarks effective operator is  $\mathcal{O}_2 = (\bar{c}\Gamma_\mu c)(\bar{s}\Gamma^\mu b)$ , with  $\Gamma_\mu = \gamma_\mu(1 - \gamma_5)$ .

The decay amplitude of the process is calculated from the Hamiltonian (3), and it can be factorized by splitting the matrix element in two pieces:

$$\mathcal{M} = i \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left( C_2 + \frac{C_1}{3} \right) \times \langle B(p) | J_\mu^W | K(p') \rangle \langle X(q) | J^{\mu(\bar{c}c)} | 0 \rangle, \quad (4)$$

where  $p = p' + q$ , and the currents are

$$J_\mu^W = \bar{s}\Gamma_\mu b, \quad J_\mu^{(\bar{c}c)} = \bar{c}\Gamma_\mu c. \quad (5)$$

The matrix elements in Eq. (4) are parametrized in the following way

$$\langle X(q) | J_\mu^{(\bar{c}c)} | 0 \rangle = \lambda_W \varepsilon_\mu^*(q), \quad (6)$$

and

$$\langle B(p) | J_\mu^W | K(p') \rangle = f_+(q^2)(p_\mu + p'_\mu) + f_-(q^2)(p_\mu - p'_\mu). \quad (7)$$

The parameter  $\lambda_W$  in (6) gives the coupling between the current  $J_\mu^{(\bar{c}c)}$  and the  $X$  state. The form factors  $f_\pm(q^2)$  describe the weak transition  $B \rightarrow K$ . Hence we can see that the factorization of the matrix element describes the decay as two separated subprocesses.

The decay width for the process  $B^\pm \rightarrow X(3872)K^\pm$  is given by

$$\Gamma(B \rightarrow XK) = \frac{1}{16\pi m_B^3} \lambda^{1/2}(m_B^2, m_K^2, m_X^2) |\mathcal{M}|^2, \quad (8)$$

with  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ . The invariant amplitude squared can be obtained from (4):

$$|\mathcal{M}|^2 = \frac{G_F^2}{2} |V_{cb} V_{cs}|^2 \left( C_2 + \frac{C_1}{3} \right)^2 \times \lambda(m_B^2, m_K^2, m_X^2) \lambda_W^2 f_+^2(q^2) |_{q^2 \rightarrow -m_X^2}. \quad (9)$$

We will use QCD sum rules in order to determine the parameter  $\lambda_W$  and the form factor  $f_+(q^2)$ , and therefore we can obtain the width of the decay.

## 3. Two-point correlator

The QCD sum rule approach [35–37] is based on the two point correlator:

$$\Pi_{\mu\nu}(q) = i \int d^4 y e^{iq \cdot y} \langle 0 | T \{ J_\mu^X(y) J_\nu^{(\bar{c}c)}(0) \} | 0 \rangle, \quad (10)$$

where the current  $J_\nu^{(\bar{c}c)}$  is defined in (5). For the  $X$  meson we will follow [31] and consider a mixed charmonium-molecular current as in Refs. [29,30]. For the charmonium part we use the conventional axial current:

$$j_\mu^{(2)}(x) = \bar{c}_a(x) \gamma_\mu \gamma_5 c_a(x). \quad (11)$$

The  $D^0 D^{*0}$  molecule is interpolated by [38–40] (with  $q = u$ ):

$$j_\mu^{(4q)}(x) = \frac{1}{\sqrt{2}} \left[ (\bar{q}_a(x) \gamma_5 c_a(x) \bar{c}_b(x) \gamma_\mu q_b(x)) - (\bar{q}_a(x) \gamma_\mu c_a(x) \bar{c}_b(x) \gamma_5 q_b(x)) \right]. \quad (12)$$

As in Ref. [31] we define the normalized two-quark current as

$$j_\mu^{(2q)} = \frac{1}{6\sqrt{2}} (\bar{u}u) j_\mu^{(2)}, \quad (13)$$

and from these two currents we build the following mixed charmonium-molecular current for the  $X(3872)$ :

$$J_\mu^q(x) = \sin(\theta) j_\mu^{(4q)}(x) + \cos(\theta) j_\mu^{(2q)}(x). \quad (14)$$

We will consider a small admixture of  $D^+D^{*-}$  and  $D^-D^{*+}$  components, then we have for the  $X$  current:

$$J_\mu^X(x) = \cos\alpha J_\mu^u(x) + \sin\alpha J_\mu^d(x), \quad (15)$$

with  $J_\mu^u(x)$  and  $J_\mu^d(x)$  given by Eq. (14).

Considering the  $u$  and  $d$  quarks to be degenerate, i.e.,  $m_u = m_d$  and  $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle$ , and inserting the currents (5) and (15) in the correlator we have in the OPE side of the sum rule

$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(q) &= (\cos\alpha + \sin\alpha) \left( \sin\theta \Pi_{\mu\nu}^{4,2}(q) \right. \\ &\quad \left. + \frac{\langle\bar{q}q\rangle}{6\sqrt{2}} \cos\theta \Pi_{\mu\nu}^{2,2}(q) \right), \end{aligned} \quad (16)$$

where

$$\begin{aligned} \Pi_{\mu\nu}^{2,4}(q) &= i \int d^4y e^{iq\cdot y} \langle 0 | T \{ J_\mu^{4q}(y) J_\nu(\bar{c}c)(0) \} | 0 \rangle, \\ \Pi_{\mu\nu}^{2,2}(q) &= i \int d^4y e^{iq\cdot y} \langle 0 | T \{ J_\mu^{2q}(y) J_\nu(\bar{c}c)(0) \} | 0 \rangle. \end{aligned} \quad (17)$$

The contribution from the vector part of the current  $J_\nu^{(\bar{c}c)}$  vanishes after the integration is performed, hence these correlators are equal (except for a minus sign) to the ones calculated in Ref. [29] for the two-point correlator of the  $X(3872)$ .

On the phenomenological side the correlator is determined inserting the intermediate state of the  $X$ :

$$\begin{aligned} \Pi_{\mu\nu}^{\text{phen}}(q) &= \frac{i}{q^2 - m_X^2} \langle 0 | J_\mu^X | X(q) \rangle \langle X(q) | J_\nu^{(\bar{c}c)} | 0 \rangle \\ &= \frac{i\lambda_X\lambda_W}{Q^2 + m_X^2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{m_X^2} \right) \end{aligned} \quad (18)$$

where  $q^2 = -Q^2$ , and we have used the definition (6) and

$$\langle 0 | J_\mu^X | X(q) \rangle = \lambda_X \epsilon_\mu(q). \quad (19)$$

The parameter defining the coupling between the current  $J_\mu^X$  and the  $X$  meson has been calculated in Ref. [29], and its value is  $\lambda_X = (3.6 \pm 0.9) \times 10^{-3} \text{ GeV}^5$ .

In the QCDSR approach a Borel transform to  $Q^2 \rightarrow M^2$  ( $Q^2 = -q^2$ ) is performed to improve the matching between both sides of the sum rules. The Borel transform exponentially suppresses the contribution from excited states in the phenomenological side of the sum rule. In the OPE side the Borel transform suppresses the contribution from higher dimension condensates [17]. After performing the Borel transform we get in the structure  $g_{\mu\nu}$ :

$$\begin{aligned} \lambda_W \lambda_X e^{-\frac{m_X^2}{M^2}} &= -(\cos\alpha + \sin\alpha) \left( \sin\theta \Pi^{24}(M^2) \right. \\ &\quad \left. + \frac{\langle\bar{q}q\rangle}{6\sqrt{2}} \cos\theta \Pi^{2,2}(M^2) \right). \end{aligned} \quad (20)$$

This expression is analysed numerically to obtain the coupling parameter  $\lambda_W$ . We perform the calculation using the same values for the masses and QCD condensates listed in [29], and in the same region in threshold parameter  $s_0$  and Borel mass  $M^2$  that we have used in the mass and  $\lambda_X$  analysis in Ref. [29],  $\sqrt{s_0} = 4.4 \text{ GeV}$ ,  $2.6 \text{ GeV}^2 \leq M^2 \leq 3.0 \text{ GeV}^2$ . The mixing angles determined in the same reference are:

$$5^\circ \leq \theta \leq 13^\circ, \quad \alpha = 20^\circ. \quad (21)$$

Taking into account the variation in the Borel mass parameter and the mixing angle  $\theta$ , the result for the  $\lambda_W$  parameter is:

$$\lambda_W = (1.29 \pm 0.51) \text{ GeV}^2. \quad (22)$$

#### 4. Three-point correlator

The form factor of the  $B \rightarrow K$  transition matrix (7) can be evaluated from the three point correlator:

$$\Pi_\mu(p, p') = \int d^4x d^4y e^{i(p'\cdot x - p\cdot y)} \langle 0 | T \{ J_\mu^W(0) J_K(x) J_B^\dagger(y) \} | 0 \rangle. \quad (23)$$

In the OPE side we use the weak current  $J_\mu^W$  defined in (5) and the interpolating currents of the  $B$  and  $K$  pseudoscalar mesons:

$$J_K(x) = i\bar{u}_a(x)\gamma_5 s_a(x), \quad J_B = i\bar{u}_a(x)\gamma_5 b_a(x). \quad (24)$$

We work at leading order in  $\alpha_s$  and consider condensates up to dimension 5 and terms linear in the mass of the  $s$  quark.

The phenomenological side of the sum rule is computed by inserting the intermediate states of the  $B$  and  $K$  mesons in the correlator:

$$\Pi_\mu^{\text{phen}} = -\frac{f_B f_K m_K^2 m_B^2}{m_b(m_s + m_u)} \frac{(f_+(t)(p + p')_\mu + f_-(t)q_\mu)}{(p^2 - m_B^2)(p'^2 - m_K^2)}, \quad (25)$$

with  $t = q^2 = (p - p')^2$ , using (7) and the following definitions:

$$\begin{aligned} \langle 0 | J_K | K(p') \rangle &= f_K \frac{m_K^2}{m_s + m_u}, \\ \langle 0 | J_B | B(p) \rangle &= f_B \frac{m_B^2}{m_b}. \end{aligned} \quad (26)$$

Performing a double Borel transform,  $p^2 \rightarrow M^2$  and  $p'^2 \rightarrow M'^2$ , and matching both sides of the sum rule, we get in the structure  $(p_\mu + p'_\mu)$  (with  $p^2 = -p'^2$ ,  $p'^2 = -p'^2$ ,  $Q^2 = -q^2 = -t$ ):

$$\begin{aligned} &-\frac{f_B f_K m_K^2 m_B^2 f_+(t)}{m_b(m_u + m_s)} e^{-\frac{m_B^2}{M^2} - \frac{m_K^2}{M'^2}} \\ &= \frac{-1}{4\pi^2} \int_{s_{\min}}^{s_0} ds \int_0^{u_0} du \rho^{\text{pert}}(s, t, u) e^{-\frac{s}{M^2} - \frac{u}{M'^2}} \\ &\quad + \frac{1}{2} \langle\bar{u}u\rangle (m_b + m_s) e^{-\frac{m_b^2}{M^2}} \\ &\quad - \frac{m_0^2 \langle\bar{u}u\rangle e^{-\frac{m_0^2}{M^2}}}{8M^4 M'^2} ((m_b^3 + m_b^2 m_s)(M^2 + M'^2) \\ &\quad - M^2(m_b(M^2 + 2M'^2 + t) + m_s(M'^2 + 2M^2 + t))), \end{aligned} \quad (27)$$

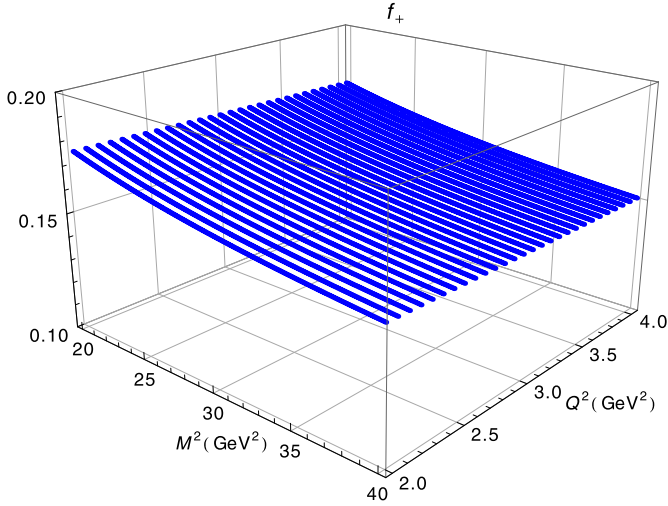
where the perturbative contribution is given by

$$\begin{aligned} \rho^{\text{pert}}(s, t, u) &= \frac{3}{4\lambda^{\frac{3}{2}}(s, t, u)} [u(2m_b^2 - s - t + u)(2m_b m_s - s - u + t) \\ &\quad + ((s - m_b^2)(s - t + u) - 2su)(2m_b m_s - s + t - u) \\ &\quad + (s + u - mb^2)\lambda(s, t, u)]. \end{aligned} \quad (28)$$

The integration limit  $s_{\min}$  is given by

$$s_{\min} = m_b^2 + \frac{m_b^2 u}{m_b^2 - t}. \quad (29)$$

$s_0 = (m_B + \Delta_s)^2$  and  $u_0 = (m_K + \Delta_u)^2$  are the continuum threshold parameters for the  $B$  and  $K$  respectively. Note that, after the double Borel transform, the contributions from the quark condensate and mixed condensates of the  $s$  quark are eliminated.



**Fig. 2.** The plot of the form factor  $f_+$  calculated as a function of  $Q^2$  and  $M^2$  from Eq. (27).

We use the following relation between the Borel masses  $M^2$  and  $M'^2$  [41]:

$$M'^2 = \frac{0.64 \text{ GeV}^2}{m_b^2 - m_b'^2} M^2. \quad (30)$$

#### 4.1. Result for the form factor

The sum rules are analysed numerically using the following values for quark masses and QCD condensates, and for meson masses and decay constants [27,42,43]:

$$\begin{aligned} m_b(m_b) &= 4.7 \text{ GeV}, & m_s &= 0.140 \text{ GeV}, \\ \langle \bar{q}q \rangle &= -(0.23 \pm 0.03)^3 \text{ GeV}^3, \\ m_0^2 &= 0.8 \text{ GeV}^2, \\ m_B &= 5.279 \text{ GeV}, & m_K &= 493.677 \text{ MeV}, \\ f_B &= 0.170 \text{ GeV}, & f_K &= 0.160 \text{ GeV}. \end{aligned} \quad (31)$$

We use the value of the mixing angles  $\alpha$  and  $\theta$  given in (21). For the continuum threshold parameters we take  $\Delta_s = \Delta_u = 0.5 \text{ GeV}$ .

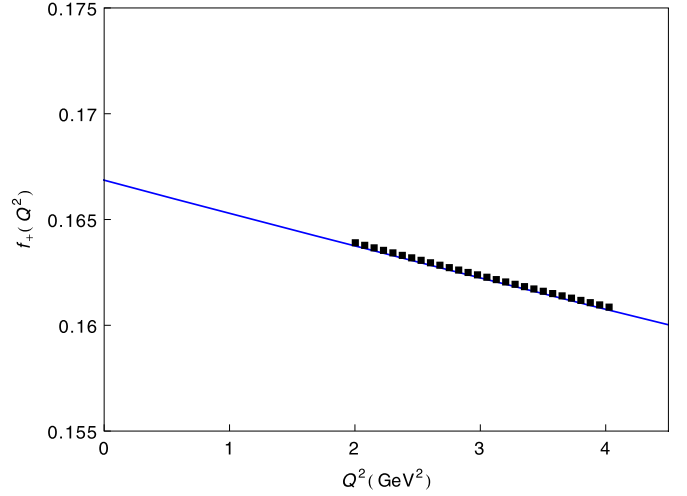
In Fig. 2 we show the plot for the form factor  $f_+$  calculated in the sum rules from Eq. (27) as a function of the Borel mass  $M^2$  and the momentum transfer  $Q^2$ . For the region  $M^2 \geq 20 \text{ GeV}^2$  shown in the plot the sum rule presents a good stability.

In the following analysis, to determine the  $Q^2$  dependence of the form factor, we choose the Borel mass within the region of stability around the mass of the  $B$  meson,  $26 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ . We fit (27) within the stable region by matching both sides of the sum rule. In Fig. 3 we show, through the dots, the QCDSR results for the form factor  $f_+(Q^2)$  as a function of  $Q^2$ . The numerical results can be well fitted by a monopolar parametrization (shown by the solid line in Fig. 3):

$$f_+(Q^2) = \frac{(17.55 \pm 0.04) \text{ GeV}^2}{(105.0 \pm 1.76) \text{ GeV}^2 + Q^2}. \quad (32)$$

For the decay width calculation, we need the value of the form factor at  $Q^2 = -m_X^2$ , where  $m_X$  is the mass of the off-shell  $X$  meson. Therefore we have:

$$f_+(Q^2)|_{Q^2=-m_X^2} = 0.195 \pm 0.003. \quad (33)$$



**Fig. 3.** Momentum dependence of  $C(Q^2)$  for  $\Delta_s = \Delta_u = 0.5 \text{ GeV}$ . The solid line gives the parametrization of the QCDSR results (dots) through (32).

#### 4.2. The decay width $B \rightarrow XK$

To determine the decay width we insert, in the expression (8), the parameter  $\lambda_W$  (22) obtained in the two-point sum rule calculation, and the value of the form factor  $f_+$ , from the three-point sum rule, at the  $X$  meson pole (33). The branching ratio is therefore calculated dividing the result by the total width of the  $B$  meson  $\Gamma_{\text{tot}} = \hbar/\tau_B$ :

$$\mathcal{B}(B \rightarrow X(3872)K) = (1.00 \pm 0.68) \times 10^{-5}, \quad (34)$$

where we have used the mean life of the  $B$  meson  $\tau_B = 1.638 \times 10^{-12} \text{ s}$ , the CKM parameters  $V_{cs} = 1.023$ ,  $V_{cb} = 40.6 \times 10^{-3}$  [43], and the Wilson coefficients  $C_1(\mu) = 1.082$ ,  $C_2(\mu) = -0.185$ , computed at  $\mu = m_b$  and  $\bar{\Lambda}_{\text{MS}} = 225 \text{ MeV}$  [44]. The result (34) is in agreement with the experimental upper limit (1).

For completeness we also compute the branching ratio for the  $X$  as pure  $\bar{c}c$  and molecular states. We choose the mixing angle as  $\theta = 90^\circ$  and  $\theta = 0^\circ$  in Eq. (14), and we get respectively for the pure molecule and pure charmonium:

$$\mathcal{B}(B \rightarrow X_{\text{mol}}K) = (0.38 \pm 0.06) \times 10^{-6}, \quad (35)$$

$$\mathcal{B}(B \rightarrow X_{\bar{c}c}K) = (2.68 \pm 0.50) \times 10^{-5}. \quad (36)$$

Comparing the results for the pure states with the one for the mixed state (34), we can see that the branching ratio for the pure molecule is one order smaller, while the pure charmonium is larger.

The result for the pure molecular state in Eq. (35) can be compared with the one from Ref. [32]. In this work the authors study the production of the  $X$  as a molecular state in the decay  $B \rightarrow XK$  through the coalescence of charm mesons. The calculation of the branching ratio is strongly dependent on the choice of parameters, and it is found to be of order  $10^{-4}$  to  $10^{-6}$ . In Ref. [34], the production of the  $X$  as a  $2^3P_1$  charmonium state is studied in pQCD, and their result for the branching ratio is  $7.88_{-3.76}^{+4.87} \times 10^{-4}$ , which is an order bigger than our result for the pure charmonium (36).

## 5. Summary and conclusions

We have presented a QCDSR analysis of the production of the  $X(3872)$  state considering a mixed charmonium-molecular current in the decay  $B \rightarrow XK$ . We find that the sum rules result in Eq. (34), obtained by using the factorization hypothesis, is smaller, but compatible with the experimental upper limit. Since, it is

known that non-factorizable contributions may play an important role in hadronic decays of  $B$  mesons [45], our result can be interpreted as a lower limit for the branching ratio.

This result was obtained by considering the mixing angles in Eqs. (15) and (14) with the values  $\alpha = 20^\circ$  and  $5^\circ \leq \theta \leq 13^\circ$ . The present result is also compatible with previous analysis of the mass of the  $X$  state and the decays into  $J/\psi\pi^0\pi^+\pi^-$  and  $J/\psi\pi^+\pi^-$  [29], and the radiative mode  $\gamma J/\psi$  [30], since the values of the mixing angles used in both calculations are the same. It is important to mention that there is no new free parameter in the present analysis and, therefore, the result presented here strengthens the conclusion reached in Refs. [29,30] that the  $X(3872)$  is probably a mixture between a  $c\bar{c}$  state and  $D^0\bar{D}^{*0}$ ,  $\bar{D}^0D^{*0}$ ,  $D^+D^{*-}$  and  $D^-D^{*+}$  molecular states. From Eq. (14) one may be tempted to say that the  $c\bar{c}$  component of the state described by such current is dominant ( $\sim 97\%$ ), like the conclusion presented in Ref. [29]. However, from a closer look at Eq. (13), one can see that the  $c\bar{c}$  component of our current is already multiplied by a dimensional parameter, the quark condensate, in order to have the same dimension of the molecular part of the current. Therefore, it is not clear that only the angle in Eq. (14) determines the percentage of each component. To try to evaluate the importance of each part of the current it is better to analyse the results obtained with each component, like the results presented in Eqs. (35) and (36). From these results we see that the  $c\bar{c}$  part of the state plays a very important role in the determination of the branching ratio. By the other hand, in the decay  $X \rightarrow J/\psi\pi^+\pi^-$  and  $X \rightarrow J/\psi\pi^+\pi^-\pi^0$ , the width obtained in our approach for a pure  $c\bar{c}$  state is [29]:

$$\Gamma(X_{c\bar{c}} \rightarrow J/\psi\pi\pi) = 0, \quad (37)$$

and, therefore, the molecular part of the state is the only one that contributes to this decay, playing an essential role in the determination of this decay width. Also, for a pure  $c\bar{c}$  state one gets:

$$M_{X_{c\bar{c}}} = (3.52 \pm 0.05) \text{ GeV}, \quad (38)$$

from where one sees again that the molecular part of the state plays a very important role in the determination of its mass.

Therefore, although we cannot determine the percentages of the  $c\bar{c}$  and the molecular components in the  $X(3872)$ , we may say that both components are extremely important, and that, in our approach, it is not possible to explain all the experimental data about the  $X(3872)$  with only one component.

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